Our previous research (Korotayev and Khaltourina, 2006: 37-44) has allowed us to suggest the following tentative overall reconstruction of the long-term Egyptian population dynamics (see Diagrams 1 and 2):

**Diagram 1:** Estimated Population Dynamics of Egypt, in thousands (10000 BCE-2005 CE)

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*I would like to express my deepest gratitude to Prof. Elizabeth Sartain, Prof. Eleonora Fernandez and Prof. Nelly Hanna of the Arabic Studies Department, American University in Cairo, for their invaluable help and advice. This research has been supported by the Russian Foundation for Basic Research (Project # 06-06-80459а), the Russian Science Support Foundation, and INTAS (Project # 05-10000008-7922).*
Diagram 2: Estimated Population Dynamics of Egypt, in thousands
(300 BCE-1900 CE)

This research has shown that the overall population trend in the 1st-18th centuries CE in Egypt was upward, rather than downward. On the other hand, the carrying capacity of land appears to have grown in Medieval Egypt considerably higher than population, whereas the population growth was significantly slower than throughout the Old World System (outside the Middle East) (Korotayev and Khaltourina, 2006: 37-56).

We believe this phenomenon could be at least partly accounted for by the mechanisms of political-demographic cycles typical for Egypt (and apparently for the other countries of the Islamic Middle East), on the one hand, and the other major regions of Eurasia on the other. Medieval Egyptian political-demographic cycles had a rather short (an order of 90 years) length. During relatively short Medieval Egyptian political-demographic cycles the population simply had not enough time to reach the carrying capacity of land ceiling. Political-demographic collapses took place well before the population reached the carrying capacity ceiling. Medieval Egypt suffered from underpopulation rather than overpopulation, the Medieval Egyptian population fluctuated well below the carrying-capacity-of-land level.
(Korotayev and Khaltourina, 2006: 45-56). Thus the political-demographic cycle models that connect demographic collapses with the ecological niche saturation and that describe rather well political-demographic dynamics of pre-Modern China (see: e.g., Nefedov, 2004) do not appear appropriate for Medieval Egypt.

This might not be a coincidence that the mathematical model that appears to describe the medieval Egyptian political-demographic better than the rest of the models, is the one (Turchin, 2003: 131-137) that was developed in an attempt to formulate in a mathematical form some part of the theory of political-demographic dynamics proposed by a person, who spent a substantial part of his life just in Medieval Egypt, ‘Abd al-Rahmān Ibn Khaldūn (1332-1406).

We believe it makes sense to start this part of the article with a summary of those points of Ibn Khaldun’s sociopolitical theory, on which Turchin’s (2003) model is based (and on which, to a considerable extent, our own versions of this model will also be based).

0. A central notion of Ibn Khaldun’s theory is ’asabiyyah (اصبابة), which will be interpreted below after Turchin (2003: 38-39) as “collective solidarity”.

1. New dynasties can be established only by groups with a very high ’asabiyyah (Ibn Khaldun, 1958, 1: 284-285, 313; 2: 119; 2004: 183-184, 201, 360).


3. This process is accounted for up to a considerable extent by the growing prestige consumption (.taraf, ترف, “luxury”) of the ruling elite (Ibn Khaldun, 1958, 1: 338-341, 343-347, 35--355; 2: 90, 123, 125-126; 2004: 216-223, 226-228, 343, 360-363).

4. Within four generations the ruling elite’s ’asabiyyah decreases to such a critical level that it leads to the dynasty (and, hence, political system) breakdown and the establishment of a new dynasty by a new high-’asabiyyah group (Ibn Khaldun, 1958, 1: 278-282, 343-346; 2004: 180-181, 220-222). Note that this suggests that a typical length of a political-demographic cycle should be between 80 and 100 years¹, which is indeed extremely close to what we could see above with respect to medieval Egyptian political-demographic cycles.

Ibn Khaldun also makes a number of other relevant observations and generalizations:

¹If we estimate a generation length as 20-25 years. Note that Ibn Khaldun himself estimated it as 40 years (1959, 1: 344, 346; 2004: 220-222).

6. However, over-taxation undermines the economy; as a result, the elites turn out to be unable to increase their revenues to a degree that would satisfy their growing demands (Ibn Khaldun, 1958, 1: 340-341; 2: 90-91, 103-104, 109-111, 136-137; 2004: 218-219, 343-344, 351-352, 354-355, 370). Ibn Khaldun connects this process directly to the dynastic decline, explaining that when the elite consumption grows faster than the state revenues, this would undermine its military force (Ibn Khaldun, 1958, 1: 340-341; 2004: 218-219). Turchin describes another way in which this process would contribute to the dynastic decline: it would lead to increased competition within the ruling elite, thus destroying the elite’s collective solidarity (‘asabiyyah) (Turchin, 2003: 132-136).

7. During early phases of political cycles a very high rate of elite reproduction is observed: “…A tribe that obtained royal authority and luxury is prolific and produces many children, and the (elite) community grows” (Ibn Khaldun, 1958, 1: 351; 2004: 225). Ibn Khaldun did not study a link between this phenomenon and the process of dynastic decline; however, Turchin argues in a very convincing way that it should have been a very important factor, as a very high rate of elite reproduction would lead to a further increase of amount of resources needed to satisfy the growing prestige consumption demands of the elite, thus increasing both the competition within the ruling elite and the over-taxation.

These points describe what could be called a theory of political cycles, but it could hardly be denoted as a theory of political-demographic cycles. However, Ibn Khaldun suggests a number of other observations and generalizations, which in conjunction with points 1-7 produce a veritable theory of political-demographic cycles:

8. Low taxation and political order observed at the early phases of dynastic cycles result in overall economic and demographic growth leading to a significant increase in numbers of not only elite members, but commoners as well (Ibn Khaldun, 1958, 2: 135; 2004: 369).

9. Over-taxation and rebellions observed during the late phases of dynastic cycles lead to the destruction of economy, famines and epidemics (directly connected with famine, state breakdown, as well as relative overpopulation) and, thus, to the demographic collapse affecting not only elites, but commoners as well (Ibn Khaldun, 1958, 2: 103-104, 109-111, 136-137; 2004: 351-352, 354-355, 370).

Turchin (2003) developed two models that he denoted by himself as “Ibn Khaldun models”, within which political-demographic collapses are produced not
by actual overpopulation, but rather by elite overpopulation, elite overproduction that can well take place in a generally underpopulated country (or at least in a country whose population is still significantly below the saturation level), and thus suggests direction within which the political-demographic dynamics of medieval Egypt could be adequately described.

However, in a few points these models fail to describe adequately the political-demographic dynamics of medieval Egypt (see Korotayev and Khaltourina, 2006 for detail).

Although Turchin’s models were inspired by Ibn Khaldun’s treatise, in the process of the model development Turchin moved rather far from the original theory of Ibn Khaldun and less of it survived in the final versions of his models. We believe that in order to produce a mathematical model describing medieval political-demographic dynamics more accurately we should try to follow Ibn Khaldun’s theory more closely. Note in particular that Turchin abstained from modeling the part of this theory described above under number 9.

We think it makes sense to reproduce at this point a full quotation of that part of *al-Muqqadimah* where the mechanisms of the political-demographic collapses are described by Ibn Khaldun in a most clear and succinct way:

“In the later (years) of dynasties, famines and pestilences become numerous. As far as famines are concerned, the reason is that most people at that time refrain from cultivating the soil. For, in the later (years) of dynasties, there occur attacks on property and tax revenue and, through customs duties, on trading. Or, trouble occurs as the result of the unrest of the subjects and the great number of rebels (who are provoked) by the senility of the dynasty to rebel. Therefore, as a rule, little grain is stored. The grain and harvest situation is not always good and stable from year to year. The amount of rainfall in the world differs by nature. The rainfall may be strong or weak, little or much. Grain, fruits, and (the amount of) milk given by animals varies correspondingly. Still, for their food requirements, people put their trust in what it is possible to store. If nothing is stored, people must expect famines. The price of grain rises. Indigent people are unable to buy any and perish. If for some years nothing is stored, hunger will be general. The large number of pestilences has its reason in the large number of famines just mentioned. Or, it has its reason in the many disturbances that result from the disintegration of the dynasty. There is much unrest and bloodshed, and plagues occur” (Ibn Khaldun, 1958, 2: 136; 2004: 370).

This text gives us very clear and convincing advice on how the model under consideration should be amended so that it could describe medieval Egyptian political-demographic dynamics more accurately. In fact Ibn Khaldun suggests that we should take into account the effect of climatic fluctuations. Needless to
say, this factor is especially relevant for the “pre-Aswan Dam” Egypt, where agricultural yields depended largely on the levels of the Nile inundations which are totally unpredictable, and vary with an extremely significant magnitude (see, e.g., Park, 1992). Fundamentally, this means that people have no way to predict the forthcoming year’s yield, but they know that sooner or later the Nile will not rise to the necessary level, resulting in a catastrophic crop failure. Within such a context, the storage of sufficient amounts of food turns out to be essential to prevent demographic collapse, and indeed the shrinking of food reserves under the influence of over-taxation (caused in its turn by elite over-population and over-consumption) could be an extremely important cause of such collapses.

In fact, the influence of this factor on the demographic cycle dynamics has already been modeled by Nefedov (2004). However, his model does not describe the actual political-demographic dynamics of medieval Egypt, as Nefedov does not take into consideration the effects of elite overpopulation that were modeled so convincingly by Turchin. Thus, in order to produce a “more Khaldunian” model of the medieval Egyptian political-demographic dynamics it appears necessary to combine respective elements of Turchin’s and Nefedov’s models into one model.

In our model the annual production ($Y$) is measured in minimum annual rations (MARs). Thus, $Y_i$, the production in year $i$, can be also regarded as $K_i$, the carrying capacity of land for this year, the maximum number of people that this annual product can support.

In our first model we assume that technological base (the technologically determined production per unit of cultivated land) and cultivated area remain constant, thus, $Y_i$, the production in year $i$, is determined entirely by climatic conditions in this year. In this model the annual production in most years fluctuates in the range that can potentially support 6.0-8.0 million people. However, once in every 10 years catastrophic crop failures are randomly observed, when the annual product turns out to be sufficient to support (even at the bare survival level) only 2.5-3.0 million people. In this model the problem of population survival in the “lean” years is solved in the following way. In a “good” year $i$ the state collects as taxes ($T_i$) half of “surplus”, $Y_i - P_{ci-1}$, that is, the yield produced over the annual amount that is absolutely necessary for bare survival of commoner population found at the beginning of year $i$ ($P_{ci-1}$) at zero reproduction level (in the “lean” years the commoners are assumed not to pay taxes). At the beginning of a cycle a quarter of the remaining surplus is stored by the population in order to procure its survival through forthcoming “lean” years.

However, this is not the only purpose for which the reserve resources are used in the lean years, as a part of elite’s demands could be also covered from this
source. In good years of the first phase of a cycle the collected taxed are divided into two parts. One part \( (C_{ei}) \) is spent to cover the elite consumption demands, whereas the other part \( (dS_i) \) is accumulated in the state treasury \( (S) \).

With Turchin we assume that the elite population grows with a faster speed due to general polygyny practiced by the elites. At the first phase of a cycle the elite consumption is provided with resources collected as taxes. If in the given year the amount of resources needed to cover the constantly growing elite demands \( (C_{ei}) \) exceeds the tax revenues of this year \( (T_i) \), no transfers to the state treasury are made \( (dS_i = 0) \), that is, all the collected taxes are consumed by the elites), whereas the deficient resources \( (C_{ei}, T_i) \) are extracted (in the form of various extortions, requisitions, and “illegitimate taxes”/mukus [مکوس], as so vividly described by Ibn Khaldun [1958, 2: 103-104, 109-111, 136-137; 2004: 351-352, 354-355, 370]) from the surplus produced by the commoners.

During the third phase of a cycle the growth of the prestige consumption of the elites leads to the situation when even in good years all the produced surplus turns out to be insufficient to cover the demands of numerous elites addicted to luxury. Now even in good years the elites extract the entire surplus produced by the commoners, the resources stop being accumulated in the counter-famine reserves; what is more, if the entire surplus extracted by the elites from the commoners turns out to be insufficient to cover the elite demands, the deficient resources are taken from the anti-famine reserves. In the lean years the anti-famine reserves are used both to prevent the decrease of commoner population and to cover the elite demands. These reserves are used more and more to cover elite demands even in good years. As a result, these reserves are depleted rather quickly. After this the elites start robbing the state treasury, and the fourth and final phase of dynastic (= political-demographic) cycle begins. During this phase even in the best years the demands of more and more numerous and luxury-addicted elites are not covered by surplus produced by the commoners. At this phase the counter-famine reserves are totally depleted (and in full accordance with Ibn Khaldun’s theory they are not replenished). The deficient resources are found by the elites in the state treasury that is also depleted in a rather rapid way.

As in the “Khaldunian” models of Peter Turchin when the state treasury is entirely depleted, the state breaks down and (in the following year) the country falls under the control of a new relatively small, “high-asabiyyah”, not corrupted by luxury elite, which is modeled by ascribing to variables \( P_e \) (elite population) and \( B \) (average per capita elite consumption) their initial values. However, within the model even before the dynastic collapse a demographic collapse takes place. Indeed, during the 4th phase the elites systematically extract the entire surplus produced by the commoners, the counter-famine reserves are absent, and the commoner population turns out to be totally defenseless in the face of any
forthcoming catastrophic crop failure. Thus, if the given annual yield is only sufficient to support, say, 2.5 million commoners, while the actual commoner population is 4 million, the commoner population will decrease precisely to 2.5 million.

In general, our basic mathematical model of medieval Egyptian political-demographic cycles can be presented in the following form (see: Table 1).
Table 1: Basic Mathematical Model of Medieval Egyptian Political-Demographic Cycles

<table>
<thead>
<tr>
<th>Variable Symbol</th>
<th>This Symbol Denotes</th>
<th>=</th>
<th>Value in Year $i$</th>
<th>If</th>
<th>Equation #</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_i$</td>
<td>Yield produced in year $i$, measured as a number of people that can be potentially supported by it at zero reproduction level, or minimum annual rations (MARs)</td>
<td>=</td>
<td>random number, mostly fluctuating in the range of 6.0–8.0 mln., but once every 10 years randomly taking values between 2.5 and 3 mln.</td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>$P_{ci}$</td>
<td>Commoner population at the end of year $i$</td>
<td>=</td>
<td>$P_{ci-1} + 0.02(1 - \frac{P_{ci-1}}{K_{ai}})P_{ci-1}$</td>
<td>$K_{ai} \geq P_{ci-1}$</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$K_{ai}$</td>
<td>$K_{ai} &lt; P_{ci-1}$</td>
<td>(3)</td>
</tr>
<tr>
<td>$K_{ai}$</td>
<td>Actual carrying capacity of land in year $i$, that is commoner population that this year pure yield (i.e., total yield minus transfers to counter-crisis reserves, as well as legal and illegal taxes and requisitions) could support at zero reproduction level</td>
<td>=</td>
<td>$Y_i - dR_j - T_i$</td>
<td>$Y \geq P_{ci-1}$ and $C_{ci} \leq T_j$</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Y_i - dR_j - C_{ei}$</td>
<td>$Y \geq P_{ci-1}$ and $C_{ci} &gt; T_j$ and $C_{ci} &lt; Y_i - P_{ci-1}$</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$P_{ci-1}$</td>
<td>$Y \geq P_{ci-1}$ and $C_{ci} \geq Y_i - P_{ci-1}$</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Y_i + (P_{ci-1} - Y_i) = P_{ci-1}$</td>
<td>$Y &lt; P_{ci-1}$ and $R &gt; P_{ci-1} - Y_i$</td>
<td>(7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Y_i$</td>
<td>$Y &lt; P_{ci-1}$ and $R_i = 0$</td>
<td>(8)</td>
</tr>
<tr>
<td>Variable Symbol</td>
<td>This Symbol Denotes</td>
<td>Value in Year $i$</td>
<td>If</td>
<td>Equation #</td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------------</td>
<td>-------------------</td>
<td>----</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>$R_i$</td>
<td>Counter-famine reserves at the end of year $i$</td>
<td>$R_{i+1} + 0.25(Y_i - P_{i+1} - T_i)$</td>
<td>$Y_i \geq P_{i+1}$ and $C_a &lt; T_i$</td>
<td>(9)</td>
<td></td>
</tr>
<tr>
<td>$R_i$</td>
<td>$R_{i+1} + 0.25(Y_i - P_{i+1} - C_a)$</td>
<td>$Y_i \geq P_{i+1}$ and $C_a \geq T_i$</td>
<td>(10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_i$</td>
<td>$R_{i+1} - (P_{c,i} - Y_i) - C_a$</td>
<td>$Y_i &lt; P_{i+1}$ and $R_{i+1} &gt; P_{i+1} - Y_i + C_a$</td>
<td>(11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_i$</td>
<td>$0$</td>
<td>$R_{i+1} &lt; P_{i+1} - Y_i + C_a$</td>
<td>(12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_a$</td>
<td>Resources consumed by the elite population in year $i$</td>
<td>$P_{e,i} B_i$</td>
<td></td>
<td>(13)</td>
<td></td>
</tr>
<tr>
<td>$P_{e,i}$</td>
<td>Elite population in year $i$</td>
<td>$P_{e,i+1} + 0.04 P_{e,i}$</td>
<td>$S_{e,i} &gt; 0$</td>
<td>(14)</td>
<td></td>
</tr>
<tr>
<td>$P_{e,i}$</td>
<td>$P_{e,i}$</td>
<td>$S_{e,i} = 0$</td>
<td>(15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_i$</td>
<td>Overall population in year $i$</td>
<td>$P_{e,i} + P_{a,i}$</td>
<td></td>
<td>(16)</td>
<td></td>
</tr>
<tr>
<td>$B_i$</td>
<td>Average per capita elite consumption in year $i$</td>
<td>$B_{i+1} + dB_i$</td>
<td>$S_{i+1} &gt; 0$</td>
<td>(17)</td>
<td></td>
</tr>
<tr>
<td>$B_i$</td>
<td>$B_0$</td>
<td>$S_{i+1} = 0$</td>
<td>(18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_i$</td>
<td>Amount of legitimate taxes collected in year $i$</td>
<td>$0.5(Y_i - P_{c,i})$</td>
<td>$P_{c,i} - Y_i &gt; 0$</td>
<td>(19)</td>
<td></td>
</tr>
<tr>
<td>$T_i$</td>
<td>$0$</td>
<td>$P_{c,i} - Y_i \leq 0$</td>
<td>(20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_i$</td>
<td>Resources accumulated in the state treasury by the end of year $i$</td>
<td>$S_{i+1} - T_i - C_a$</td>
<td>$C_a \leq T_i$</td>
<td>(21)</td>
<td></td>
</tr>
<tr>
<td>$S_i$</td>
<td>$S_{i+1}$</td>
<td>$C_a &gt; T_i$ and $C_a \leq R_{i+1}$</td>
<td>(22)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_i$</td>
<td>$S_{i+1} - (C_a - R_{i+1})$</td>
<td>$C_a &gt; T_i$ and $C_a &gt; R_{i+1}$ and $C_a &lt; S_{i+1}$</td>
<td>(23)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_i$</td>
<td>$0$</td>
<td>$C_a &gt; T_i$ and $C_a &lt; R_{i+1}$ and $C_a \geq S_{i+1}$</td>
<td>(24)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Typical dynamics generated by this model look as follows (See: Diagram 3).

**Diagram 3:** Typical Dynamics Generated by the Basic Model of Medieval Egyptian Political-Demographic Cycles

(a) Dynamics of Overall Population (black curve, thousands) and Counter-Famine Reserves (grey curve, tens of thousands of MARs)

(b) Dynamics of Elite Population (black curve, thousands) and Resources Accumulated by the State (grey curve, hundreds of thousands of MAR equivalents)

This model provides a relatively adequate description of the political-demographic dynamics of medieval Egypt in all its basic characteristics (especially up to 1347): political-demographic cycles of approximately 90-year length, the reduction of population growth during the last phases of a cycle, precipitous depopulations in the course of political-demographic collapses, and the absence of pronounced intercycles whereby a rather steady recovery growth starts immediately (or almost immediately) after the demographic collapse.
Special attention is deserved by the fact that, as in Turchin’s extended model, in our basic model the increase of the natural elite growth rate leads to the decline of the political-demographic cycle length, whereas its decrease results in the lengthening of those cycles.

Thus, it turns out to be possible to produce the model version describing rather adequately the basic features of medieval European political-demographic dynamics through the decrease by 4 times of the “Egyptian” natural elite growth rate coefficient (which would correspond to the strictly monogamous reproduction context typical for all the medieval European Christians, including the elites, see: Diagram 4).

**Diagram 4:** Typical Dynamics Generated by the Basic Model of Medieval European Political-Demographic Cycles

As we see, this version of the model describes adequately the medieval European political-demographic macrodynamics in its basic characteristics: c.200-year-long political-demographic cycles and pronounced intercycles (see, e.g., McEvedy and Jones, 1978; Turchin, 2005). Special attention should be paid to the fact that within the “European” version of the model population systematically approaches the carrying capacity of the land much closer than in the “Egyptian” version.

One of the main simplifying assumptions of our basic model is that the technologically determined carrying capacity is considered to be constant. In reality, of course, it was not a constant, but a variable with a pronounced long-term upward trend dynamics. This trend is conditioned by technological innovations whose intensity also tends to grow (see, e.g., Korotayev, Malkov, and Khaltourina, 2006). This way we treat this variable in our extended model, and it allows us to investigate numerically the influence of the “secular cycle” structure on the “millennial” economic and demographic trends.
Our extended model of medieval political-demographic and economic-technological dynamics differs from the basic model with the two following points:

1. Instead of the climatically determined annual yield ($Y_i$) in the extended model we take into account the annual production ($A_i$), determined not only by the climatic conditions of the given year, but also by technological development level (index) achieved by the beginning of the given year ($I_{i-1}$).

2. With Kremer (1993), Cohen (1995), Podlazov (2004), and Tsirel (2004) we use the following technological growth equation:

$$\frac{dI}{dt} = aPI,$$

where $I$ denotes technological development index, and $P$ is population.

Our extended model also takes into account the “Boserupian” effect. Ester Boserup (1965) has shown that the relative overpopulation creates powerful stimuli to generate and introduce innovations that raise the carrying capacity. Indeed, if there is no land shortage, the population has no strong incentives to generate and/or apply such innovations, whereas under conditions of relative overpopulation such incentives become really strong, as the introduction of such innovations becomes literally a question of “life or death” for a substantial part of the population, and the intensity of generation and introduction of the carrying capacity increasing innovations significantly grows. In our model the Boserupian assumption is modeled by a two-fold increase of the coefficient value in the technological growth equation when the ecological niche gets filled more than 50%.

Typical dynamics generated by the extended model are represented below (See.: Diagram 5).

**Diagram 5**: Typical Dynamics Generated by the Extended Mathematical Model of Medieval Political-Demographic Cycles
(a 700-year computer simulation)
NOTE to Diagram 5: thick grey line – overall population dynamics within the “Egyptian” (polygynous) version of the model (in thousands); thin grey line – average carrying capacity dynamics within the “Egyptian” (polygynous) version of the model (in thousands of people); thick black line – overall population dynamics within the “European” (monogamous) version of the model (in thousands); thin black line – average carrying capacity dynamics within the “European” (monogamous) version of the model (in thousands of people).

Hence, our numerical investigation of this model suggests that within the “European” (“monogamous”) version of the model subsistence technologies do tend to develop faster than those within the “Egyptian” (“polygynous”) version. Thus, in a 700-year simulation the technological development index grew by more than 63% within the first version and constituted less than 47% in the second. At the meantime a comparison of population at cycle peaks indicates that within the “Egyptian” model population tends to approach the ceiling of the carrying capacity of land to a much smaller degree than is observed within the “European” version of the model. It is highly remarkable that within the “Egyptian” model a significant growth of the carrying capacity could exist without a parallel demographic growth. Moreover, for considerable periods of time it can be accompanied by a certain population decline, which appears to have been actually observed for certain parts of the medieval Egyptian history. The contrast between trajectories of the countries developing according to these two models will look especially salient if we continue the period of computer simulation twice, up to 1400 years (See: Diagram 6).

Diagram 6: Typical Dynamics Generated by the Extended Mathematical Model of Medieval Political-Demographic Cycles (a 1400-year computer simulation)
NOTE to Diagram 6: thick grey line – overall population dynamics within the “Egyptian” (polygynous) version of the model (in thousands); thin grey line – average carrying capacity dynamics within the “Egyptian” (polygynous) version of the model (in thousands of people); thick black line – overall population dynamics within the “European” (monogamous) version of the model (in thousands); thin black line – average carrying capacity dynamics within the “European” (monogamous) version of the model (in thousands of people).

By the end of this period the technological development in a country evolving according to the “European” model exceeds by almost 5 times the technological level of a country that develops according to the “Egyptian” model. What is more, within the first model by the end of period in question demographic collapses disappear, the rate of economic growth starts to systematically outstrip the population growth rate. Thus, the per capita production begins to grow with accelerating speed and the system begins in a rather confident way its escape from the “Malthusian trap“, within which the population of the country evolving according to the “polygynous” model continues to remain (See: Diagram 7).

Diagram 7: Typical Dynamics of Per Capita Output (in MARs) Generated by the Extended Mathematical Model of Medieval Political-Demographic cycles (years 1350-1650 of the computer simulation, roughly corresponding to the 16th -18th centuries in real time)

Mathematical Model of Medieval Political-Demographic Cycles (years 1350-1650 of the computer simulation, roughly corresponding to the 16th -18th centuries in real time)
The closest fit with the actually observed long-term political-demographic dynamics of Egypt in the 1\textsuperscript{st}-18\textsuperscript{th} centuries CE is observed when we bring our model closer to the Egyptian history realities and take into consideration the fact that during the period covered by the present study we observe the transition from strictly monogamous elites to those practicing polygyny in a rather extensive way.

In the last version of our extended model (the “monogamous-polygynous” one) this is modeled in the following way: the initial value of the elite natural growth coefficient in equation (14) is taken to be equal to 0.01, and after one of the cycles, at the moment corresponding to the power in the country being seized by a new, polygynous, elite its value increase 4-fold up to 0.04, which corresponds to the transfer of power from monogamous elites to polygynous ones. Typical dynamics generated by the “monogamous-polygynous” version of the extended model of medieval political-demographic cycles is represented below (See: Diagram 8)

\textbf{Diagram 8:} Typical Dynamics Generated by the “Monogamous-Polygynous” Version of Extended Mathematical Model of Medieval Political-Demographic Cycles.

\begin{center}
\includegraphics[width=\textwidth]{diagram8.png}
\end{center}

\textbf{NOTE to Diagram 8:} \textit{thick black line} – overall population dynamics (in thousands); \textit{thin black line} – average carrying capacity dynamics (in thousands of people).

As we see, the model provides a rather adequate mathematical description of the phenomenon that was detected earlier (Korotayev and Khaltourina 2006) through the analysis of the economic-demographic dynamics of Egypt in the 1\textsuperscript{st}-
18th centuries, when a rather significant increase in the carrying capacity of land was accompanied by a comparatively insignificant population growth. Thus, our models suggest possible ways to account for this phenomenon.

We can also see that millennial trend dynamics are very closely connected with the structure of secular cycles, whereas the structure of millennial trends cannot be adequately understood without secular cycles being taken into account.

Note also that our analysis suggests that pre-industrial Christian Europe was characterized by rates of demographic and technological growth that were significantly higher than in the Islamic Middle East to some extent because of the strict monogamy imposed and maintained by the Christian Church in an effective manner throughout the whole population, including the elites, as well as some other norms and practices imposed by the Church, whose application led to a significant decrease of the natural elite population growth. In addition to the strictest possible prohibition of polygyny, already in the 4th century the Christian Church imposed the regulations which prohibited close marriages, discouraged adoption, condemned concubinage, divorce and remarriage. As has been suggested by Goody (1983: 44-46), in this way the Church appears to have striven towards obtaining the property left by couples lacking legitimate male heirs. Note, however, that among other things those norms contributed in a rather significant way to the bringing down of the natural elite population growth rates in the Christian world, and, thus affected secular cycle structure in a very serious way.

Last but not least, we wish to refer to an important medieval Christian institution of clergy celibacy. This institution contributed to the blocking of the elite overproduction in two important ways. On the one hand, the absence of legitimate natural heirs of the members of the Church elite meant that for every generation a very large number of elite positions became open within the Church system, which alleviated significantly the problem of the elite overproduction, as very large numbers of excessive elites members could be canalized to the Church system, as they had an opportunity to make a socially approved and acceptable carrier within the Church hierarchy. On the other hand, these excessive elite members having found themselves within the Church hierarchy were effectively taken out of the process of the natural reproduction of the elite that contributed in a rather significant way to the decrease of the rate of the natural elite population growth.

Hence, norms and practices imposed by the Church led to a significant decrease of the natural elite population growth and affected the secular cycle structure in a way that contributed to the situation when Europe was characterized by rates of
demographic and technological growth that were significantly higher than in the Islamic Middle East.

REFERENCES


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